7.3 Volume By Washers

$$V_{washer} = \pi\left((\text{outer radius})^2 - (\text{inner radius})^2\right)(\text{thickness})$$

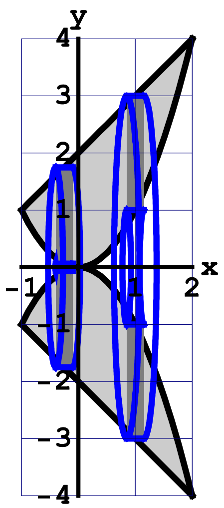
x – axis $\quad V_{washer} = \pi \int_a^b \left((f(x))^2 - (g(x))^2\right) dx \quad$ where $f(x)$ is the outer radius and $g(x)$ is the inner radius

y – axis $\quad V_{washer} = \pi \int_c^d \left((f(y))^2 - (g(y))^2\right) dy \quad$ where $f(y)$ is the outer radius and $g(y)$ is the inner radius

For problems 1 – 8, find the volume of the indicated solid.

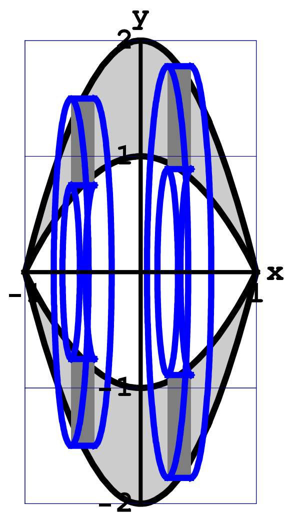
1. $y = x + 2$, $y = x^2$, rotated around the x – axis

$$V_{washers} = \pi \int_{-1}^{2} \left((x+2)^2 - (x^2)^2\right) dx$$



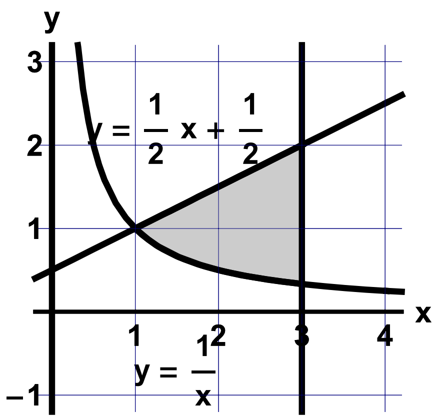
2. $y = 2 - 2x^2$, $y = 1 - x^2$, rotated around the x – axis

$$V_{washers} = 2\pi \int_{0}^{1} \left((2 - 2x^2)^2 - (1 - x^2)^2\right) dx$$



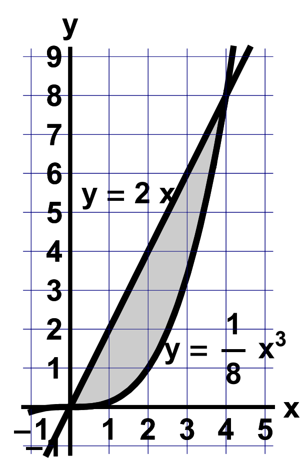
3. $y = \frac{1}{2}x + \frac{1}{2}$, $y = \frac{1}{x}$, and $x = 3$, rotated around the x – axis

$$V_{washers} = \pi \int_{1}^{3} \left(\left(\frac{1}{2}x + \frac{1}{2}\right)^2 - \left(\frac{1}{x}\right)^2\right) dx$$



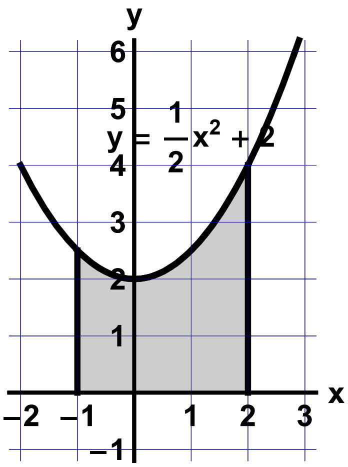
4. $y = 2x$, $y = \frac{1}{8}x^3$, rotated around the y – axis

$$V_{washers} = \pi \int_{0}^{8} \left(\left(2y^{\frac{1}{3}}\right)^2 - \left(\frac{1}{2}y\right)^2\right) dy$$



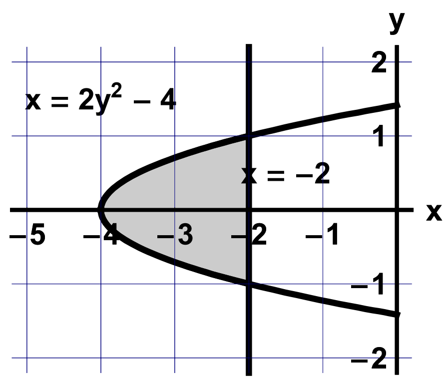
5. $y = \frac{1}{2}x^2 + 2, \quad y = 0, \quad x = -1, \quad x = 2,$ rotated around the x – axis

$$V_{washers} = \pi \int_{-1}^{2} \left(\left(\frac{1}{2}x^2 + 2 \right)^2 - 0^2 \right) dx$$



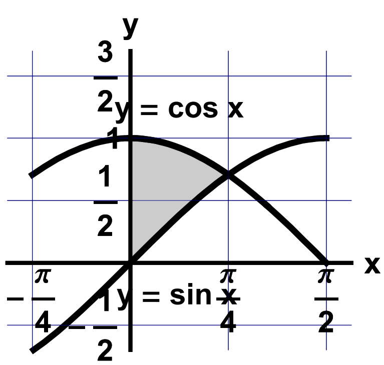
6. $x = 2y^2 - 4, \quad x = -2,$ rotated around the y – axis

$$V_{washers} = 2\pi \int_{0}^{1} \left((4 - 2y^2)^2 - 2^2 \right) dy$$



7. $y = \cos x, \quad y = \sin x, \quad x = 0, \quad x = \frac{\pi}{4},$ rotated around the x – axis

$$V_{washers} = \pi \int_{0}^{\frac{\pi}{4}} \left((\cos x)^2 - (\sin x)^2 \right) dx$$



8. $x = y^{\frac{2}{3}}, \quad x - y = 2, \quad \text{and} \quad y = 8,$ rotated around the y – axis

$$V_{washers} = \pi \int_{-1}^{8} \left((2 + y)^2 - \left(y^{\frac{2}{3}} \right)^2 \right) dy$$