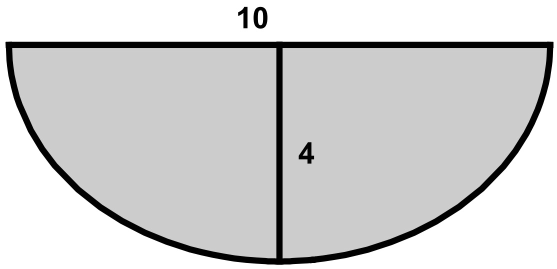
7.5 Applications from Science and Statistics

Finding the force of a fluid on a plate submerged in a liquid

$F = \int_c^d \rho L(y)\, h(y)\, dy$ where ρ is the density of the fluid, $L(y)$ is the length of a function representing the length of the plate, and $h(y)$ represents the depth.

1. The vertical end of a tank containing water with a density of $62.5 \frac{\text{lbs}}{\text{ft}^3}$ is shown. Find the force of the water on one end of the tank.

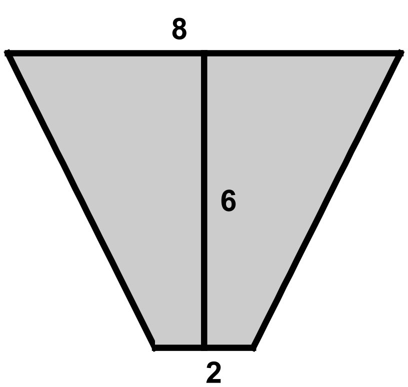


$L(y) = ?$ $\quad \frac{x^2}{5^2} + \frac{y^2}{4^2} = 1 \quad x^2 = 25 - \frac{25y^2}{16} \quad x = \frac{\pm 5}{4}\sqrt{16 - y^2} \quad L(y) = \frac{5}{2}\sqrt{16 - y^2}$

$h(y) = (-y) \quad F = 62.5 \int_{-4}^{0} \frac{5}{2}\sqrt{16 - y^2}\,(-y)\,dy \quad u = 16 - y^2 \quad du = -2y\,dy$ or $\frac{1}{2}du = y\,dy$ so

$F = 62.5\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)\int_0^{16} u^{\frac{1}{2}}\,du = \left(\frac{125}{2}\right)\left(\frac{5}{4}\right)\left(\frac{2}{3}\right)\left[u^{\frac{3}{2}}\right]_0^{16} = \frac{625}{12}(16)^{\frac{3}{2}} = \frac{625}{12}(64) = 625\left(\frac{16}{3}\right) = \boxed{\frac{1000}{3} \text{ lbs}}$

2. The vertical end of a tank containing water with a density of $62.5 \frac{\text{lbs}}{\text{ft}^3}$ is shown. Find the force of the water on one end of the tank.



$L(y) = ?$ when $y = 0$, $L(0) = 2$ when $y = 6$, $L(6) = 8$ $m = \dfrac{8-2}{6-0} = 1$

$L(y) = y + 2$ $h(y) = 6 - y$ $F = 62.5\displaystyle\int_0^6 (y+2)(6-y)\,dy = \frac{125}{2}\int_0^6 (12 + 4y - y^2)\,dy$ so

$$F = \frac{125}{2}\left[12y + 2y^2 - \frac{y^3}{3}\right]_0^6 =$$

$$\frac{125}{2}\left(72 + 72 - \frac{216}{3}\right) = \frac{125}{2}\left(\frac{216}{3} + \frac{216}{3} - \frac{216}{3}\right) \quad \text{so}$$

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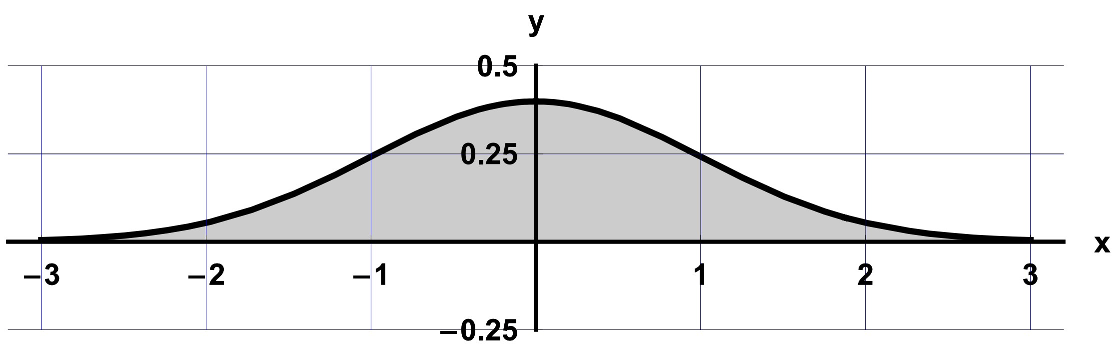
$$F = \frac{125}{2}\left(\frac{216}{3}\right) = 125(36) = \boxed{4500 \text{ lbs}}$$

The normal probability density function for a population with mean μ and standard deviation σ is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$ for this function,

$\displaystyle\int_{\mu-\sigma}^{\mu+\sigma} f(x)\,dx = 0.68,$ $\displaystyle\int_{\mu-2\sigma}^{\mu+2\sigma} f(x)\,dx = 0.95,$ $\displaystyle\int_{\mu-3\sigma}^{\mu+3\sigma} f(x)\,dx = 0.997,$ and $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1$



3. The mean gpa at Monta Vista is 2.9 with a standard deviation of 0.7. What proportion of students at Monta Vista:
   (a) Have gpa′s less than 2.9?
   (b) Have gpa′s between 3.5 and 3.9?
   (c) Have gpa′s less than 2.0?
   (d) Have gpa′s off exactly 3.1?

(a) 0.5 (b) $\displaystyle\int_{3.5}^{3.9} \frac{1}{0.7\sqrt{2\pi}}\, e^{\frac{-(x-2.9)^2}{2(0.7)^2}}\,dx$ (c) $\displaystyle\int_{0.1}^{2.0} \frac{1}{0.7\sqrt{2\pi}}\, e^{\frac{-(x-2.9)^2}{2(0.7)^2}}\,dx$

(d) $\displaystyle\int_{3.1}^{3.1} \frac{1}{0.7\sqrt{2\pi}}\, e^{\frac{-(x-2.9)^2}{2(0.7)^2}}\,dx = 0$

4. The mean score on the SAT at Monta Vista is 1210 with a standard deviation of 180. What proportion of MV students scored:

(a) above 1390?

(b) between 800 and 1100?

(c) greater than 1420?

(d) exactly 1300?

(a) $\dfrac{(1 - 0.68)}{2} = 0.16$ (b) $\displaystyle\int_{800}^{1100} \frac{1}{180\sqrt{2\pi}} e^{\frac{-(x-1210)^2}{2(180)^2}}\,dx$ (c) $\displaystyle\int_{1420}^{1930} \frac{1}{180\sqrt{2\pi}} e^{\frac{-(x-1210)^2}{2(180)^2}}\,dx$

(d) $\displaystyle\int_{1300}^{1300} \frac{1}{180\sqrt{2\pi}} e^{\frac{-(x-1210)^2}{2(180)^2}}\,dx = 0$

Work = change in kinetic energy, or $W = \frac{1}{2} m v^2$ when the body that is in motion starts at rest

Remember that: weight = mass x acceleration, so

Newtons are measured in kilograms x $\dfrac{\text{meters}}{\text{sec}^2}$ and pounds are measured in slugs x $\dfrac{\text{feet}}{\text{sec}^2}$

5. A 2 ounce tennis ball is served at a speed of 160 $\dfrac{\text{ft}}{\text{sec}}$ (about 109 mph). How much work was done on the ball to make it go that fast?

$$2 \text{ ounces} \times \frac{1\text{ lb}}{16 \text{ ounces}} \times \frac{1}{32\ \frac{\text{ft}}{\text{sec}^2}} = \frac{1}{256}\ \frac{\text{lbs}}{\text{ft}/\text{sec}^2} \qquad \text{so}$$

$$\text{Work} = \frac{1}{2}\left(\frac{1}{256}\ \frac{\text{lbs}}{\text{ft}/\text{sec}^2}\right)\left(160\ \frac{\text{ft}}{\text{sec}}\right)^2 = \boxed{50 \text{ lb ft}}$$