7.5 Work

Solve problems 1 – 5.

1. Find the work done by the force of $F(x)$ Newtons along the x – axis from $x = a$ to $x = b$, where
   $F(x) = \sin(\pi x)$ $a = 0$, $b = 1$

$$W = Fd \quad \rightarrow \quad W = \int_a^b F(x)\,dx = \int_0^1 \sin(\pi x)\,dx = \frac{-1}{\pi}\left[\cos(\pi x)\right]_0^1 = \frac{-1}{\pi}(-1 - 1) = \boxed{\frac{2}{\pi}}$$

2. An elevator weighing 3000 lbs. is suspended by a cable weighing 14 $\frac{\text{lbs.}}{\text{foot}}$. Find the work done in lifting the elevator (and its cable) 180 feet.

$$W_{approx} = (3000\text{ lbs.})(180\text{ ft.}) +$$
$$(180\text{ feet})\left(14\,\frac{\text{lbs}}{\text{foot}}\right)(1\text{ foot}) + (179\text{ feet})\left(14\,\frac{\text{lbs}}{\text{ft}}\right)(1\text{ foot}) + (178\text{ feet})\left(14\,\frac{\text{lbs}}{\text{ft}}\right)(1\text{ foot})$$
$$+ (177\text{ feet})\left(14\,\frac{\text{lbs}}{\text{ft}}\right)(1\text{ foot}) + \ldots + (1\text{ feet})\left(14\,\frac{\text{lbs}}{\text{ft}}\right)(1\text{ foot}) \quad \text{so}$$

$$\boxed{W = (3000\text{ lbs.})(180\text{ ft.}) + \int_0^{180} (180 - y)(14)\,dy}$$

and the result would be given in ft – lbs

3. A fish tank is 5 feet tall, 4 feet long, and 3 feet wide. If water weighs 62.5 $\frac{\text{lbs.}}{\text{ft}^3}$, and the tank is initially filled with water, then find the work done in lifting all of the water out of the tank. Then, find the work done if the water is lifted 6 feet above the top of the tank. Then, find the work done if the tank is only partially filled with water, to a height of 2 feet.

$$W_{approx} = \left(3 \times 4 \times \frac{1}{12}\text{ ft}^3\right)\left(62.5\,\frac{\text{lbs}}{\text{ft}^3}\right)(5\text{ feet}) + \left(3 \times 4 \times \frac{1}{12}\text{ ft}^3\right)\left(62.5\,\frac{\text{lbs}}{\text{ft}^3}\right)\left(4\,\frac{11}{12}\text{ feet}\right)$$
$$+ \left(3 \times 4 \times \frac{1}{12}\text{ ft}^3\right)\left(62.5\,\frac{\text{lbs}}{\text{ft}^3}\right)\left(4\,\frac{5}{6}\text{ feet}\right) + \left(3 \times 4 \times \frac{1}{12}\text{ ft}^3\right)\left(62.5\,\frac{\text{lbs}}{\text{ft}^3}\right)\left(4\,\frac{3}{4}\text{ feet}\right)$$
$$+ \ldots + \left(3 \times 4 \times \frac{1}{12}\text{ ft}^3\right)\left(62.5\,\frac{\text{lbs}}{\text{ft}^3}\right)\left(\frac{1}{12}\text{ feet}\right) \quad \text{so}$$

$$\boxed{W = \int_0^5 (3 \times 4)\,62.5\,(5 - y)\,dy}$$ and

$$\boxed{W = \int_0^5 (3 \times 4)\,62.5\,(11 - y)\,dy}$$ when the water is lifted 6 feet above the top of the tank, and

$$\boxed{W = \int_0^2 (3 \times 4)\,62.5\,(5 - y)\,dy}$$ when the water is only filled to a height of 2 feet

4. A bucket is to be lifted 12 feet. The bucket weighs 2 lbs., and it is initially filled with 4 lbs. of water. However, water is leaking from the bucket at a constant rate, so that only 1 lb. of water remains after its trip of 12 feet. Find the work done in lifting the bucket and water.

total weight at the bottom → 6 lbs.   total weight at the top → 3 lbs.   so

when $y = 0 \rightarrow f(y) = 6$ when $y = 12 \rightarrow f(y) \rightarrow 3$ so

$$m = \frac{3 - 6}{12 - 0} = \frac{-1}{4} \quad \text{and} \quad f(y) = 6 - \frac{1}{4}y \quad \rightarrow \quad \boxed{W = \int_0^{12} \left(6 - \frac{1}{4}y\right) dy}$$

5. A spring has a natural length of 6 inches. If a force of 10 lbs. is required to stretch the spring to a length of 12 inches, then find the work done in stretching the spring from a length of 6 inches to a length of 14 inches.

$$f(x) = kx \quad \rightarrow \quad 10 = k(6) \quad \rightarrow \quad k = \frac{5}{3} \qquad f(x) = \frac{5}{3}x \quad \text{and} \quad \boxed{W = \int_0^8 \frac{5}{3}x\,dx}$$