8.1 L′Hopital′s Rule

Suppose f and g are differentiable on an open interval (a, b) containing c, except possibly at c itself. If $\frac{f(x)}{g(x)}$ has the indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ at $x = c$ and if $g'(x) \neq 0$ for $x \neq c$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}, \quad \text{provided either} \quad \lim_{x \to c} \frac{f'(x)}{g'(x)} \quad \text{exists} \quad \text{or} \quad \lim_{x \to c} \frac{f'(x)}{g'(x)} = \infty$$

This rule will allow us to find the limit of a variety of forms, including:

$$\frac{0}{0},\ \frac{\infty}{\infty},\ \infty \cdot 0,\ \infty - \infty,\ 1^{\infty},\ 0^{0},\ \infty^{0}$$

Solve problems 1 – 10.

1. $\lim_{x \to 0} \frac{\cos x + 4x - 1}{3x}$

2. $\lim_{x \to 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)}$

3. $\lim_{x \to \infty} \frac{e^x}{x^2 + x}$

4. $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$

5. $\lim_{x \to \infty} x \ln\left(\frac{x - 1}{x + 1}\right)$

6. $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

7. $\lim_{x \to \infty} \left(\sqrt{x^2 + 5x} - x \right)$

8. $\lim_{x \to 0^+} (1 + 2x)^{\frac{1}{3x}}$

9. $\lim_{x \to 1^+} (x - 1)^{\ln x}$

10. $\lim_{x \to \infty} (\ln x)^{\frac{1}{x}}$