8.2 Relative Rates of Growth

COMPARING RATES OF GROWTH

Let $f(x)$ and $g(x)$ be positive for x sufficiently large

1. f grows faster than g as $x \to \infty$ if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty$

2. f grows slower than g as $x \to \infty$ if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$

3. f grows at the same rate as g as $x \to \infty$ if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = L \neq 0$, where L is some real number

LITTLE – OH

Let $f(x)$ and $g(x)$ be positive for x sufficiently large

Then f is of smaller order than g as $x \to \infty$ if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$ and $f = o(g)$

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Let $f(x)$ and $g(x)$ be positive for x sufficiently large

Then f is of at most the order of g as $x \to \infty$ if there is a positive integer M for which $\frac{f(x)}{g(x)} \leq M$ for x sufficiently large, and $f = O(g)$

For problems 1 – 6, determine if $f(x)$ grows faster than, or slower than, or at the same rate as $g(x)$

1. $f(x) = \log_3 x, \quad g(x) = \ln x^2$

2. $f(x) = \left(\frac{x}{2}\right)^2, \quad g(x) = 10x - 1$

3. $f(x) = 2^x, \quad g(x) = e^x$

4. $f(x) = 3^{2x}, \quad g(x) = 3^x$

5. $f(x) = \log \sqrt{x}, \quad g(x) = \ln x$

6. $f(x) = x - \ln x^2, \quad g(x) = \log_4 x$

7. Order the following functions from slowest – growing to fastest – growing as $x \to \infty$, 2^x, x^x, $(\ln x)^x$, 2^{2x}

For problems 8 – 15, determine whether the statement is true or false as $x \to \infty$

8. $x = o(3x)$

9. $x + \sin x = O\left(\frac{x}{2}\right)$

10. $4^x = O\left(4^{\frac{x}{2}}\right)$

11. $\sqrt{x}\,\log x = o(x)$

12. $2^x + x^2 = O(2^x)$

13. $\frac{3}{x} = o(\ln x)$

14. $\sqrt{x^2 + x} = O(2x)$

15. $\ln(2x) = O((\ln x)^2)$