8.3 Comparison Tests

1. Consider $\int_0^1 \frac{1}{x^p}\,dx$

A. Converges for $0 < p < 1$
B. Diverges for $p \geq 1$

2. Consider $\int_1^\infty \frac{1}{x^p}\,dx$

A. Diverges for $0 < p \leq 1$
B. Converges for $p > 1$

DIRECT COMPARISON TEST

Let f and g be continuous on $[a, \infty)$ with $0 \leq f(x) \leq g(x)$ for all $x \geq a$. Then

1. $\int_a^\infty f(x)\,dx$ converges if $\int_a^\infty g(x)\,dx$ converges

2. $\int_a^\infty g(x)\,dx$ diverges if $\int_a^\infty f(x)\,dx$ diverges

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LIMIT COMPARISON TEST

If the positive functions f and g are continuous on $[a, \infty)$ and if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = L, \quad 0 < L < \infty$

Then $\int_a^\infty f(x)\,dx$ and $\int_a^\infty g(x)\,dx$ both converge or both diverge

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For problems 1 – 8, use integration, the direct comparison test, or the limit comparison test to determine whether the integral converges or diverges.

1. $\int_1^\infty \frac{3 + \cos x}{x^3}\,dx$

2. $\int_8^\infty \frac{1}{\sqrt[3]{x} - 1}\,dx$

3. $\displaystyle\int_{-\infty}^{0} \frac{1}{x^2}\, dx$

4. $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt[4]{x} + \cos x}\, dx$

5. $\displaystyle\int_{-1}^{\infty} \left(\frac{1}{x + 1} - \frac{1}{x + 3}\right) dx$

6. $\displaystyle\int_{e}^{\infty} \frac{2}{x - \ln x}\, dx$

7. $\displaystyle\int_{-\infty}^{0} \frac{4}{\sqrt{x^6 + 2}}\, dx$

8. $\displaystyle\int_{0}^{\infty} x^2 e^{-x}\, dx$