8.3 Improper Integrals

IMPROPER INTEGRALS WITH INFINITE INTEGRATION LIMITS

1. If $f(x)$ is continuous on $[a, \infty)$, then $\displaystyle\int_a^{\infty} f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx$

2. If $f(x)$ is continuous on $(-\infty, b]$, then $\displaystyle\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx$

3. If $f(x)$ is continuous on $(-\infty, \infty)$, then $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$

   where c is any real number

IMPROPER INTEGRALS WITH INFINITE DISCONTINUITIES

1. If $f(x)$ is continuous on $(a, b]$, then $\displaystyle\int_a^b f(x)\,dx = \lim_{c \to a^+} \int_c^b f(x)\,dx$

2. If $f(x)$ is continuous on $[a, b)$, then $\displaystyle\int_a^b f(x)\,dx = \lim_{c \to b^-} \int_a^c f(x)\,dx$

3. If $f(x)$ is continuous on $[a, c) \cup (c, b]$, then $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

For problems 1 – 8, evaluate the integral or state that it diverges

1. $\displaystyle\int_{-\infty}^{0} \frac{1}{\sqrt[3]{x-1}}\,dx$

2. $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

3. $\int_0^{\infty} \frac{x}{x^4 + 1}\, dx$

4. $\int_{-2}^{0} \frac{x}{\sqrt{4 - x^2}}\, dx$

5. $\int_{-3}^{1} \frac{1}{x}\, dx$

6. $\int_{-\infty}^{\infty} x e^{-x^2}\, dx$

7. $\int_0^{\infty} \frac{\cos x}{1 + \sin^2 x}\, dx$

8. $\int_0^{\frac{\pi}{2}} \frac{1}{1 - \cos x}\, dx$