

8.4 Partial Fractions

Partial Fraction Decomposition for $\frac{f(x)}{g(x)}$

1. If the degree of $g(x)$ is less than or equal to the degree of $f(x)$, use long division
2. Express $g(x)$ in its most compact factored form
3. For each factor $(ax + b)^n$, the partial fraction decomposition contains a sum of n partial fractions of the form

$$\frac{A_1}{(ax + b)} + \frac{A_2}{(ax + b)^2} + \dots + \frac{A_n}{(ax + b)^n}$$

For each factor $(ax^2 + bx + c)^n$, the partial fraction decomposition contains a sum of n partial fractions of the form

$$\frac{A_1x + B_1}{(ax^2 + bx + c)} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \dots + \frac{A_nx + B_n}{(ax^2 + bx + c)^n}$$

Let's set one up $\rightarrow \int \frac{3x^2 + 7x - 11}{x^3(x^2 + 5)^3(x - 1)^2(x^2 + x + 1)(x + 3)^2} dx$

For problems 1 – 6, evaluate the integral.

1. $\int \frac{4x^2 + 13x - 9}{x^3 + 2x^2 - 3x} dx$

2. $\int \frac{x^2 - x - 21}{2x^3 - x^2 + 8x - 4} dx$

$$3. \int \frac{5x^3 - 3x^2 + 7x - 3}{(x^2 + 1)^2} dx$$

$$4. \int \frac{-19x^2 + 50x - 25}{x^2(3x - 5)} dx$$

$$5. \int \frac{x^3 + 6x^2 + 3x + 16}{x^3 + 4x} dx$$

$$6. \int \frac{4x^3 - 3x^2 + 6x - 27}{x^4 + 9x^2} dx$$