Chapter 8 Test **No Calculators** **February 13, 2012** **Name**

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1. Evaluate $\lim_{x \to 0^+} \cot(3x) \sin^{-1}\left(\frac{2x}{5}\right)$

$\to \infty \cdot 0 \to \lim_{x \to 0^+} \frac{\sin^{-1}\left(\frac{2x}{5}\right)}{\tan(3x)} \to \frac{0}{0} \to L' \to \lim_{x \to 0^+} \frac{\frac{1}{\sqrt{1-\left(\frac{2x}{5}\right)^2}} \frac{2}{5}}{\sec^2(3x) \cdot 3} = \boxed{\frac{2}{15}}$

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2. Evaluate $\lim_{x \to 0^+} \frac{2e^{3x} - e^{-x}}{3x}$

$\to \frac{2-1}{0^+} \to \frac{1}{0^+} \to \boxed{\infty \quad \text{Diverges}}$

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3. Evaluate $\lim_{x \to 1} \frac{3 - 3x + 3\ln x}{\cos(2\pi x) - 1}$

$\to \frac{3 - 3 + 0}{1 - 1} \to \frac{0}{0} \to L' \to \lim_{x \to 1} \frac{-3 + \frac{3}{x}}{-2\pi \sin(2\pi x)} \to \quad \to \frac{0}{0} \to L'$

$\to \lim_{x \to 1} \frac{\frac{-3}{x^2}}{-4\pi^2 \cos(2\pi x)} = \frac{-3}{-4\pi^2} = \boxed{\frac{3}{4\pi^2}}$

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4. Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{2x}\right)^{6x}$

$\to 1^\infty \to \text{New} \to \lim_{x \to \infty} 6x \ln\left(1 + \frac{1}{2x}\right) \to \infty \cdot 0 \to L' \to \lim_{x \to \infty} \frac{6\ln\left(1 + \frac{1}{2} \cdot \frac{1}{x}\right)}{\frac{1}{x}}$

$\to \frac{0}{0} \to L' \to \lim_{x \to \infty} \frac{\frac{6}{1 + \frac{1}{2x}} \frac{-1}{2x^2}}{-\frac{1}{x^2}} \to \lim_{x \to \infty} \frac{6x^2}{2x^2 + x} = 3 \to \boxed{e^3}$

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5. A circular plate of radius 3 feet is partially submerged in water, to a height 2 feet above the center of the plate. If the water has a specific weight of $62.5 \ \frac{\text{lbs}}{\text{ft}^3}$ write an integral representing the force of the fluid on one side of the plate. Do not evaluate this integral.

$x^2 + y^2 = 9 \to x = \sqrt{9 - y^2} \to 2x = 2\sqrt{9 - y^2}$

$\boxed{F = 62.5 \int_{-3}^{2} 2\sqrt{9 - y^2}\,(2 - y)\, dy} = 125\left[\int_{-3}^{2} 2\sqrt{9 - y^2}\, dy - \int_{-3}^{2} y\sqrt{9 - y^2}\, dy\right]$

First integral : This is just the area of a sector of a circle and a triangle so

$$\int_{-3}^{2} 2\sqrt{9 - y^2}\, dy = 2\left[\frac{1}{4}(\pi 3^2) + \frac{\sin^{-1}\left(\frac{2}{3}\right)}{2\pi}(\pi 3^2) + \frac{2\cdot\sqrt{3^2 - 2^2}}{2} \right] = 2\left(\frac{9\pi}{4} + \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right) + \sqrt{5} \right)$$

$$= \frac{9\pi}{2} + 9\sin^{-1}\left(\frac{2}{3}\right) + 2\sqrt{5}$$

Second integral : $\int_{-3}^{2} y\sqrt{9 - y^2}\, dy$ $\quad u = 9 - y^2$ and $du = -2y \rightarrow y = -\frac{1}{2}du$

$$= \int_{0}^{5} -\frac{1}{2}\sqrt{u}\, du = -\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}} \right]_0^5 = -\frac{1}{3}\left(5\sqrt{5} - 0\right) = \frac{-5\sqrt{5}}{3}$$

So $F = 125\left[\frac{9\pi}{2} + 9\sin^{-1}\left(\frac{2}{3}\right) + 2\sqrt{5} + \frac{5\sqrt{5}}{3} \right] = \boxed{125\left(\frac{11\sqrt{5}}{3} + \frac{9\pi}{2} + 9\sin^{-1}\left(\frac{2}{3}\right) \right)}$

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6. It's rumored that, for his lecture tomorrow, DeRuiter will wear a cape and be suspended from the ceiling (that's one straaaange dude!). Assume that he weighs 190 pounds, and the cable suspending him weighs 3 pounds per linear foot. If the winch (or pulley) is suspended from the ceiling at a point 10 feet above the ground, find the work done in lifting him to a point 6 feet above the ground.

$$6\cdot 190 + \int_{0}^{6} 3(10 - y)\, dy = 1140 \text{ ft}\cdot\text{lbs} + \int_{0}^{6} (30 - 3y)\, dy = 1140 + \left[30y - \frac{3}{2}y^2 \right]_0^6$$

$$= 1140 + 180 - \frac{3}{2}\cdot 36 = 1140 + 180 - 54 = \boxed{1266 \text{ ft}\cdot\text{lbs}}$$

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7. Evaluate $\int_{0}^{\infty} (\cos x)\left(2^{-x}\right) dx$

$u = 2^{-x}$ $\quad du = -(\ln 2)2^{-x}dx$ $\quad v = \sin x$ $\quad dv = \cos x\, dx$

$= (\sin x)2^{-x} + (\ln 2)\int(\sin x)\, 2^{-x}\, dx \rightarrow u = 2^{-x}$ $\quad du = -(\ln 2)2^{-x}dx$ $\quad v = -\cos x$ $\quad dv = \sin x\, dx$

$= (\sin x)2^{-x} + (\ln 2)\left[(-\cos x)2^{-x} - (\ln 2)\int(\cos x)\, 2^{-x}\, dx \right] \rightarrow A = (\sin x)2^{-x} + (\ln 2)\left[(-\cos x)2^{-x} - (\ln 2)A \right]$

$\rightarrow A = \frac{1}{1 + (\ln 2)^2}\left[(\sin x)2^{-x} - (\ln 2)(\cos x)2^{-x} \right] + C$ so $\rightarrow \lim_{b\to\infty} \frac{1}{1 + (\ln 2)^2}\left[(\sin x)2^{-x} - (\ln 2)(\cos x)2^{-x} \right]_0^b$

$\rightarrow 0 - \frac{1}{1 + (\ln 2)^2}(0\cdot 1 - (\ln 2)(1)(1)) = \boxed{\frac{\ln 2}{1 + (\ln 2)^2} \text{ so Converges}}$

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8. Evaluate $\int_{-\infty}^{0} \frac{3}{x^2 - 5x + 4}\, dx$

$$\rightarrow \int \frac{3}{(x - 4)(x - 1)}\, dx = \int\left(\frac{A}{x - 4} + \frac{B}{x - 1} \right) dx$$

$3 = A(x - 1) + B(x - 4)$

$x = 1 \rightarrow -3B = 3 \rightarrow B = -1$ and $x = 4 \rightarrow 3A = 3 \rightarrow A = 1$

$$\rightarrow \lim_{b\to-\infty} \int_{b}^{0}\left(\frac{1}{x - 4} - \frac{1}{x - 1} \right) dx = \lim_{b\to-\infty}\left[\ln\left| \frac{x - 4}{x - 1} \right| \right]_b^0 = \ln 4 - \lim_{b\to-\infty}\left[\ln\left| \frac{b - 4}{b - 1} \right| \right]$$

$= \boxed{\ln(4) \text{ so Converges}}$

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9. Evaluate $\int_0^1 \frac{2x + 3}{\sqrt{1 - x^2}}\,dx$

$$= \lim_{b \to 1^-} \int_0^b \frac{2x}{\sqrt{1 - x^2}}\,dx + \lim_{b \to 1^-} \int_0^b \frac{3}{\sqrt{1 - x^2}}\,dx \rightarrow u = 1 - x^2 \text{ and } -du = 2x\,dx$$

$$\rightarrow \lim_{b \to 1^-} \int_1^{1-b^2} -u^{\frac{-1}{2}}\,du + \lim_{b \to 1^-} \left[3\sin^{-1}x\right]_0^b = -2\lim_{b \to 1^-} \left[u^{\frac{1}{2}}\right]_1^{1-b^2} + 3\left(\frac{\pi}{2}\right)$$

$$= -2(0 - 1) + \frac{3\pi}{2} = \boxed{2 + \frac{3\pi}{2} \text{ so Converges}}$$

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10. Use the Direct Comparison Test or the Limit Comparison Test to determine if the following integral Converges or Diverges.

Be sure to choose a related, simpler, p – value integral for comparison. $\int_0^3 \frac{2 + x}{x^{\frac{9}{5}}}\,dx$

DIRECT : If we want to show divergence, we need to find an integral whose integrand (function) is

smaller, simpler, related, and diverges $\rightarrow \frac{2 + x}{x^{\frac{9}{5}}} \geq \frac{2}{x^{\frac{9}{5}}}$ and $2\int_0^1 \frac{1}{x^{\frac{9}{5}}}\,dx$ Diverges

so original **DIVERGES**

LIMIT : Compare to $\frac{2}{x^{\frac{9}{5}}}$ $\lim_{x \to \infty} \frac{\frac{2 + x}{x^{\frac{9}{5}}}}{\frac{2}{x^{\frac{9}{5}}}} \rightarrow 1$ so both **DIVERGE**

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11. Evaluate $\int_{-3}^0 \frac{1}{(x + 1)^{\frac{5}{3}}}\,dx$

$$= \int_{-3}^{-1} \frac{1}{(x + 1)^{\frac{5}{3}}}\,dx + \int_{-1}^{0} \frac{1}{(x + 1)^{\frac{5}{3}}}\,dx \rightarrow \lim_{b \to -1^-} \left[\frac{-3}{2}(x + 1)^{\frac{-2}{3}}\right]_{-3}^b + \lim_{b \to -1^+} \left[\frac{-3}{2}(x + 1)^{\frac{-2}{3}}\right]_b^0$$

$$\rightarrow \lim_{b \to -1^-} \left(\frac{-3}{2}\frac{1}{(b + 1)^{\frac{2}{3}}}\right) + \frac{3}{2}\cdot\frac{1}{\sqrt[3]{4}} + \lim_{b \to -1^+} \left(\frac{3}{2}\frac{1}{(b + 1)^{\frac{2}{3}}}\right) - \frac{3}{2} \rightarrow \boxed{-\infty \text{ OR } \infty \text{ so Diverges}}$$

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12. Evaluate $\int \frac{2x^3 - x^2 + 2x - 4}{x^4 + x^2}\,dx$

$$= \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}\right)dx$$

$2x^3 - x^2 + 2x - 4 = Ax(x^2 + 1) + B(x^2 + 1) + (Cx + D)x^2$

$x = 0 \rightarrow -4 = B$

$2 = A + C$

$-1 = B + D \rightarrow D = 3$

$2 = A \rightarrow C = 0$

$$\rightarrow \int \left(\frac{2}{x} - \frac{4}{x^2} + \frac{3}{x^2 + 1}\right)dx = \boxed{2\ln|x| + \frac{4}{x} + 3\tan^{-1}x + C}$$
