9.1 Differentiation and Integration of Power Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{where} \quad -1 < x < 1$$

Series Differentiation

If $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$

Then $f'(x) = \sum_{n=1}^{\infty} n\, c_n (x-a)^{n-1}$

Series Integration

If $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$

Then $\int_a^x f(t)\, dt = \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$

For problems 1 and 2, express as a ratio of two integers.

1. .863636363...

2. .3251251251...

For problems 3 – 10, find a power series to represent the given function, and the interval of convergence.

3. $\frac{2}{1+5x}$

4. $\frac{x^2}{1+(x+3)}$

5. $\ln(1 + x)$

6. $\dfrac{3}{1 + x^2}$

7. $\tan^{-1} x$

8. $\dfrac{1}{4 - 3x}$

9. $\ln(x)$

10. $\dfrac{1}{(1 - x)^2}$