9.1 Power Series

Infinite Series

$a_1 + a_2 + a_3 + \ldots + a_n + \ldots$ or $\sum_{k=1}^{\infty} a_k$

Geometric Series

$a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} + \ldots$ or $\sum_{n=1}^{\infty} ar^{n-1}$

I. Converges for $|r| < 1$, with a sum of $\frac{a}{1-r}$

II. Diverges if $|r| \geq 1$

Power Series I

$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots + c_n x^n + \ldots$ is a power series centered at $x = 0$

Power Series II

$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + c_3 (x-a)^3 + \ldots + c_n (x-a)^n + \ldots$

is a power series centered at $x = a$

For problems 1 – 3, rewrite using summation notation.

1. $\frac{2}{3} - \frac{2}{6} + \frac{2}{9} - \frac{2}{12} + \frac{2}{15} - \ldots$

2. $71 + 0.71 + 0.0071 + 0.000071 + \ldots$

3. $\frac{\pi}{2} + \frac{\pi^2}{4} + \frac{\pi^3}{8} + \frac{\pi^4}{16} + \ldots$

For problems 4 – 7, determine whether the series converges or diverges. If it converges, give its sum.

4. $1 - \frac{3}{4} + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^4 - \ldots$

5. $\sum_{n=1}^{\infty} \sin\left(\frac{\pi}{2} n\right)$

6. $\sum_{k=0}^{\infty} \left(\frac{6}{5}\right)\left(\frac{1}{3}\right)^k$

7. $0.4 - 0.04 + 0.004 - 0.0004 + \ldots$

For problems 8 – 10, find the interval of convergence and the function of x represented by the series.

8. $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{3^n}$

9. $\sum_{n=0}^{\infty} 4(2x+1)^n$

10. $\sum_{n=0}^{\infty} \frac{-(x-4)^n}{5^n}$

11. If $\sum_{n=1}^{\infty} a r^{n-1} = 7$ and $a = 3$, find r

12. Find a power series for $\frac{1}{1-x}$