9.2 Taylor Series Continued

For problems 1 – 4, find the Taylor polynomial for the given function, of the given order n.

1. $f(x) = \sin x, \ n = 4, \ a = \dfrac{\pi}{6}$

2. $f(x) = e^{2x}, \ n = 3, \ a = 1$

3. $f(x) = \ln(x), \ n = 4, \ a = e$

4. $f(x) = \tan^{-1} x, \ n = 2, \ a = 1$

5. Let f be a function that has derivatives for all orders for all real numbers. Assume that $f(2) = 3$, $f'(2) = -5$, $f''(2) = -1$, and $f^{(3)}(2) = 8$.
   (a) Write the third order Taylor polynomial for f at $x = 2$ and use it to approximate $f(1.5)$
   (b) Write the second order Taylor polynomial for f' at $x = 2$ and use it to approximate $f'(1.5)$

6. The Maclaurin series for $f(x)$ is

$$f(x) = -\frac{x}{2!} + \frac{x^3}{4!} - \frac{x^5}{6!} + \ . \ . \ . + (-1)^n \frac{x^{2n-1}}{(2n)!} + \ . \ . \ . \qquad \text{(where n begins at 1)}$$

(a) Find $f'(0)$, $f^{(4)}(0)$, and $f^{(7)}(0)$

(b) Find a function description for $f(x)$

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7. Let $f(t) = \dfrac{4}{1 - t^3}$ and $G(x) = \displaystyle\int_0^x f(t)\,dt$

(a) Find the first four terms and the general term for the Maclaurin series generated by $f(t)$

(b) Find the first four terms and the general term for the Maclaurin series generated by $G(x)$

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8. Let $P_4(x) = 5 - 2(x - 3) + 4(x - 3)^2 - 7(x - 3)^3 - 3(x - 3)^4$ be the Taylor polynomial of order 4 for the function at $x = 3$. Assume that f has derivatives of all orders for all real numbers

(a) Find $f(3)$, $f^{(2)}(3)$, and $f^{(4)}(3)$

(b) Write the second order Taylor polynomial for f'' at $x = 3$

(c) Write the fourth order Taylor polynomial for $g(x) = \displaystyle\int_3^x f(t)\,dt$ at $x = 3$