9.2 Taylor Series

Maclaurin Series

Let $f(x)$ be a function that has derivatives of all orders on an open interval containing 0. Then $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$

and the partial sum $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k$ is the Maclaurin polynomial of order n.

Taylor Series

Let $f(x)$ be a function that has derivatives of all orders on an open interval containing a. Then $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k$

and the partial sum $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x-a)^k$ is the Taylor polynomial of order n for f at $x = a$.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for} \quad -1 < x < 1$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \quad \text{for} \quad -1 < x < 1$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad \text{for} \quad -\infty < x < \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad \text{for} \quad -\infty < x < \infty$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for} \quad -\infty < x < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \quad \text{for} \quad -1 < x \le 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad \text{for} \quad -1 \le x \le 1$$

Assuming that $f(x)$ has derivatives of all orders, and that $f(x) = \sum_{n=0}^{\infty} a_n x^n$, derive the Maclaurin Series

(we're trying to find a formula to replace the a_n)

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \dots$ so $f(0)$

$f'(x)$

$f''(x)$

$f'''(x)$

$f^{(4)}(x)$

$f^{(5)}(x)$

Now, let's derive the Maclaurin Series for ________

For problems 1 – 5, use the table of Maclaurin Series given on the other side of this page. Find the Maclaurin Series for the given function and determine the interval of convergence.

1. $\cos 3x$

2. $\tan^{-1}\left(\frac{x}{4}\right)$

3. $x^2 e^{2x}$

4. $\ln\left(1 - \frac{x^2}{2}\right)$

5. $\frac{2x^3}{1 - 5x}$