

9.4 Radius of Convergence

The n^{th} – Term Test for Divergence

Consider $\sum_{n=1}^{\infty} a_n$, if

- (i) $\lim_{n \rightarrow \infty} a_n \neq 0$, then the series diverges
- (ii) $\lim_{n \rightarrow \infty} a_n = 0$, then there is no conclusion

The Ratio Test (to be used with n^{th} powers and factorials)

Let $\sum a_n$ be a series with positive terms,

and with $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = L$, Then

- (a) the series converges if $L < 1$,
- (b) the series diverges if $L > 1$,
- (c) the test is inconclusive if $L = 1$

Geometric Series

$$a + ar + ar^2 + ar^3 + \dots + ar^{n-1} + \dots \quad \text{or} \quad \sum_{n=1}^{\infty} ar^{n-1}$$

I. Converges for $|r| < 1$, with a sum of $\frac{a}{1-r}$

II. Diverges if $|r| \geq 1$

For problems 1 – 6, determine whether the given series converges or diverges. Indicate the test(s) that you use.

1. $\sum_{n=1}^{\infty} \frac{2^{n-1}}{n(3^n)}$

2. $\sum_{n=1}^{\infty} \frac{2n}{7n+3}$

3. $\sum_{n=1}^{\infty} \frac{n^2 + n}{5^n}$

4. $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{4^n}$

5. $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

6. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

The Convergence Theorem for Power Series

There are 3 possibilities for $\sum_{n=0}^{\infty} c_n (x - a)^n$ with respect to convergence

- (1) There is a positive number R such that the series diverges for $|x - a| > R$ but converges for $|x - a| < R$. The series may or may not converge at either of the endpoints $x = a - R$ and $x = a + R$
- (2) The series converges for every x ($R = \infty$)
- (3) The series converges at $x = a$ and diverges elsewhere ($R = 0$)

Absolute Convergence Implies Convergence

If $\sum |a_n|$ converges, then $\sum a_n$ converges. Also, if $\sum |a_n|$ converges, then we say that $\sum a_n$ converges absolutely

For problems 7 – 12, determine the radius of convergence of the power series.

7.
$$\sum_{n=0}^{\infty} \frac{10^n}{3^{2n}} x^n$$

8.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n} x^{2n+1}$$

9.
$$\sum_{n=0}^{\infty} \frac{2^n}{(2n)!} x^{2n}$$

10.
$$\sum_{n=1}^{\infty} \frac{2}{n(3^n)} (x - 2)^n$$

11.
$$\sum_{n=1}^{\infty} \frac{n!}{2^{3n}} (x + 1)^n$$

12.
$$\sum_{n=1}^{\infty} \frac{\ln n}{e^n} (x + e)^n$$