9.5 Testing Convergence at Endpoints

For problems 1 – 8, determine the interval of convergence. For this interval, find where the power series converges conditionally, and where it converges absolutely.

1. $\sum_{n=0}^{\infty} (x+1)^n$

2. $\sum_{n=1}^{\infty} \frac{n x^{2n}}{4^n}$

3. $\sum_{n=1}^{\infty} \frac{3^{2n}(x-4)^n}{n!}$

4. $\sum_{n=2}^{\infty} \frac{\left(x-\frac{\pi}{2}\right)^n}{2^n \sqrt{n^3}}$

5. $\sum_{n=0}^{\infty} (-3)^n (2x+3)^n$

6. $\sum_{n=1}^{\infty} \frac{x^{2n+1}}{n^2+n}$

7. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}\,(x+2)^n}{3^n}$

8. $\sum_{n=2}^{\infty} \frac{(3x+1)^n}{\sqrt{n}+3}$

Alternating Series Truncation Error

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} u_n$ converges, then the truncation error for the n^{th} partial sum is less than u_{n+1} and has the same sign as the first unused term.

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9. $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ Find a bound for the error when approximating $\sin\left(\frac{\pi}{6}\right)$ with the 8^{th} degree Maclaurin polynomial. Is the approximation too big or too small?

10. $\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$ Find a bound for the error when approximating $\tan^{-1}(23)$ with the 5^{th} degree Maclaurin polynomial. Is the approximation too big or too small?