Chapter 9 Test March 13, 2012 No Calculators Name

1. Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Justify your answer

$$\frac{(3)}{(2!)} - \frac{(3\cdot 6)}{(4!)} + \frac{(3\cdot 6\cdot 9)}{(6!)} - \frac{(3\cdot 6\cdot 9\cdot 12)}{(8!)} + \cdots + \frac{(-1)^{n+1}(3\cdot 6\cdot 9\cdot 12\cdot\cdots\cdot(3n))}{(2n)!} + \ldots \quad \text{or} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3\cdot 6\cdot 9\cdot\cdots\cdot(3n))}{(2n)!}$$

2. Use the Direct Comparison Test (not the Limit Comparison Test) to determine if the following series converges or diverges

$$\sum_{n=3}^{\infty} \frac{(\log_9 3)^n + \sqrt{4n} + 100}{3\ln n + (0.9)^n + \left(\frac{1}{2}n\right)^2}$$

3. Let f be a function that has derivatives of all orders for all real numbers. Assume that $f(5) = 3$, $f'(5) = -2$, $f''(5) = 6$, $f'''(5) = -18$, $f^{(4)}(5) = -48$, and $f^{(5)}(5) = -24$.

(a) Write the fourth order Taylor polynomial for f at $x = 5$

(b) Write the fourth order Taylor polynomial for f' at $x = 5$ (that's f', the first derivative)

(c) Write the third order Taylor polynomial for $g(x) = \int_5^x f(t)\,dt$ at $x = 5$

4. **Find the interval of convergence for :** $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n+1}}{3^n}(x+2)^n$

5. **Find the interval of convergence for :** $\sum_{n=1}^{\infty} \frac{n+1}{2^{(2n+1)}}(x-3)^{2n}$

6. **Find the Maclaurin series for** $f(x) = x\cos^2(3x)$

(**Hint : Use a trig identity first, then a memorized power series . . . You may need to leave one term outside the series**)

7. Use the Integral Test to determine if the following series is convergent or divergent: $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$

8. Find $P_n(x)$ and $R_n(x)$ for $f(x) = \sin^2\left(\frac{x}{2}\right)$ at $c = \frac{\pi}{2}$, $n = 3$. (Hint: Trig Identities are our friends!)

9. The Maclaurin series for $f(x)$ is defined as $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{2n}{(n+1)!}x^n$

(a) Find $f^{(4)}(0)$ **(b) Let $g(x) = x f'(x)$. Write the Maclaurin series for $g(x)$**

(c) Let $h(x) = \displaystyle\int_0^x f(t)\,dt$. Write the Maclaurin series for $h(x)$

10. Find $P_n(x)$ for $f(x) = \dfrac{2}{2 - x}$ at $c = 1$, $n = 2$. Then, find the Lagrange Error (Remainder) when $x = \dfrac{3}{2}$

11. Find a power series expansion for $f(x) = \displaystyle\int_0^x \ln(1 + 2t)\, dt$

12. Use the Nth – Term Test to determine if the following series converges or diverges, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n + 3)^{\frac{2}{n}}}$