Chapter 9 Test March 14, 2014 No Calculators Name

1. Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Justify your answer

$$\sum_{n=3}^{\infty} \frac{(-1)^n n}{(\ln n)^3}$$

2. Use the <u>Limit</u> <u>Comparison</u> <u>Test</u> (<u>not</u> the Direct Comparison Test) to determine if the following series converges or diverges

Do not skip steps in your work. $$\sum_{n=1}^{\infty} \frac{6n^2 + 4n + 7}{\sqrt{4n^5 + n^4 + 5n^2}}$$

3. Find the Taylor Series for $f(x) = x^3 + 2x^2 - 5x - 3$, centered at $c = 2$

4. Find the interval of convergence for: $\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{(n+1)\,3^n}} (x-4)^n$

5. Find the interval of convergence for: $\sum_{n=1}^{\infty} (-2)^n \left(n^2+1\right) (x+1)^{2n+1}$

6. Find the Maclaurin series for $f(x) = x \frac{2x}{\left(1+x^2\right)^2}$ (Hint: Use a memorized power series, and this is a calculus class)

7. Use the Integral Test to verify that the following series converges for all $p > 1$. Do not skip steps in your work. $\sum_{n=1}^{\infty} \frac{1}{n^p}$

8. Find $P_n(x)$ and $R_n(x)$ for $f(x) = \sin^{-1}(x)$ at $c = \frac{1}{2}$, $n = 2$

9. Let $P_4(x) = 5 - 3(x-2) + 10(x-2)^2 + 8(x-2)^3 - 6(x-2)^4$ be the fourth degree Taylor polynomial for $f(x)$ centered at $x = 2$.

(a) Find $f''(2)$ and $f^{(4)}(2)$

(b) Find the third order Taylor polynomial for $g(x) = \int_2^x f(t)\,dt$ centered at $x = 2$

(c) Find the second order Taylor polynomial for $f''(x)$ centered at $x = 2$

10. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{(n^2+1) n!}$ converges. What is the maximum error of the fourth partial sum? Is the fourth partial sum greater than or less than the sum of the series? Justify your answers.

11. Find a power series expansion for $f(x) = \int_0^x \frac{\ln(1+t)}{t}\, dt$

12. Determine whether the following series is convergent or divergent. Justify your answer $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$