Tests For Convergence And / Or Divergence

The n^{th} – Term Test

Consider $\sum a_n$, if

(i) $\lim_{n \to \infty} a_n \neq 0$, then

the series diverges

(ii) $\lim_{n \to \infty} a_n = 0$, then

there is no conclusion

Ratio Test

Consider $\sum a_n$, if

$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$, then

(i) if $L < 1$, then

the series converges

(ii) if $L > 1$, then

the series diverges

(iii) if $L = 1$, then ??

Geometric

Consider $\sum_{n=1}^{\infty} a r^{n-1}$

(i) if $|r| < 1$,

series converges

to $S = \frac{a}{1 - r}$

(ii) if $|r| \geq 1$,

series diverges

Direct Comparison

Consider $\sum a_n$,

all terms non – negative

(i) $\sum a_n$ converges if

there is a convergent

series $\sum c_n$ such that

$a_n \leq c_n$ for all $n > N$,

for some integer N

(ii) $\sum a_n$ diverges if

there is a divergent

series $\sum d_n$ such that

$a_n \geq d_n$ for all $n > N$,

for some integer N

Limit Comparison

Suppose $a_n > 0$, $b_n > 0$,

for all $n \geq N$

(i) if $\lim_{n \to \infty} \frac{a_n}{b_n} = c$,

$0 < c < \infty$, then

$\sum a_n$ and $\sum b_n$ both

converge or both

diverge

(ii) if $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$

and $\sum b_n$ converges,

then $\sum a_n$ converges

(iii) if $\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$

and $\sum b_n$ diverges,

then $\sum a_n$ diverges

Integral Test

Consider $\sum a_n$

If $a_n = f(n)$,

where $f(n)$ is a

continuous, positive,

decreasing function

for all $x \geq N$

(N a positive int),

then $\sum_{n=N}^{\infty} a_n$ and

$\int_N^{\infty} f(x)\,dx$ either

both converge or

both diverge

p – Series

Consider $\sum_{n=1}^{\infty} \frac{1}{n^p}$

(i) if $p > 1$, then

the series converges

(ii) if $p \leq 1$, then

the series diverges

Alternating Series

Consider $\sum_{n=0}^{\infty} (-1)^n a_n$

series converges if

(i) each a_n is positive

(ii) $a_n \geq a_{n+1}$ for all

$n \geq N$ (for some int N)

(iii) $\lim_{n \to \infty} a_n = 0$

Alternating Series Estimation

if $\sum_{n=0}^{\infty} (-1)^n a_n$

converges, then the

truncation error of

the n^{th} partial sum

is less than a_{n+1}