10.1 Parametric Functions

First and Second Derivatives

If the equations $x = f(t)$ and $y = g(t)$ define y as a twice – differentiable function of x, then at any point where $\frac{dx}{dt} \neq 0$,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{dy'}{dx} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}}$$

Arc Length Formula

If a smooth curve $x = f(t)$ and $y = g(t)$ for $a \le t \le b$ is traversed exactly once as t increases from a to b, the curve's length is

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

For problems 1 – 4, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t.

1. $x = t - t^2, \; y = t - t^3$

2. $x = e^t, \; y = e^{-2t}$

3. $x = 3t^2 - 6t, \; y = \sqrt{t}$

4. $x = \cos^3 t, \; y = \sin^3 t$

For problems 5 and 6, find the points on the curve at which the tangent line is (a) horizontal, and (b) vertical.

5. $x = 2 + \cos t,\ y = -1 + \sin t$

6. $x = 2 - t,\ y = t^3 - 4t$

For problems 7 – 9, find the length of the curve on the given interval.

7. $x = 5t^2,\ y = 2t^3,\quad 0 \le t \le 1$

8. $x = e^t \cos t,\ y = e^t \sin t,\quad 0 \le t \le \dfrac{\pi}{2}$

9. $x = \cos(2t),\ y = \sin^2 t,\quad 0 \le t \le \pi$