10.2 Vectors in the Plane

Vector Component Form

If $\mathbf{v}$ is a vector in the plane equal to the vector with intitial point $(0, 0)$ and terminal point (v_1, v_2), then the component form of $\mathbf{v}$ is $\mathbf{v} = \langle v_1, v_2 \rangle$

Vector Equality

Vectors are equal if they have the same length and direction.

Vector Magnitude

The magnitude (length) of the the vector $\mathbf{v} = \langle v_1, v_2 \rangle$ is $|\mathbf{v}| = \sqrt{(v_1)^2 + (v_2)^2}$

Dot Product

$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta = u_1 v_1 + u_2 v_2$ (where θ is the angle between the two vectors)

Angle Between Two Vectors

The angle between two vectors $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is given by

$$\theta = \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right) = \cos^{-1}\left(\frac{u_1 v_1 + u_2 v_2}{|\mathbf{u}||\mathbf{v}|}\right)$$

For problems $1 - 4$, let $\mathbf{u} = \langle 4, -3 \rangle$ and $\mathbf{v} = \langle -3, 5 \rangle$. Find the (a) component form and (b) magnitude of the vector.

1. $\mathbf{u} - \mathbf{v}$

2. $3\mathbf{u} + 2\mathbf{v}$

3. $4\mathbf{v} - \mathbf{u}$

4. $\frac{1}{2}\mathbf{v} - \frac{3}{2}\mathbf{u}$

For problems 5 – 7, find the component form of the vector.

5. The sum of $\overrightarrow{AB}$ and $\overrightarrow{CD}$ where $A = (0, -2)$, $B = (-3, 5)$, $C = (6, -1)$, and $D = (-4, 2)$.

6. The unit vector with the same direction as $\overrightarrow{PQ}$, where $P = (-2, 1)$, and $Q = (3, -3)$.

7. The unit vector obtained by rotating the vector $< 0, 1 >$ 150° in a counterclockwise direction, about the origin.

For problems 8 and 9, find the unit vectors (four of them) that are tangent and normal to the curve at the given point.

8. $x = \ln(t - 1)$, $y = t - 1$, and $t = 3$

9. $x = 2\sin t$, $y = 2\cos t$, and $t = \dfrac{-\pi}{4}$